

# Robust Parameter Design with Feedback Control

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## Abstract

Two commonly used methodologies for mitigating the effect of noise on process output are robust parameter design and on-line control. In many processes, the optimal control law depends on the parameter design solution and vice-versa. The need for an integrated approach that combines the two methods is therefore evident. In this paper, a parameter design methodology in the presence of feedback control is developed for processes of long duration. Systems that follow a pure-gain dynamic model are considered and the best proportional-integral (PI) and minimum mean squared error (MMSE) control strategies are developed by using robust parameter design. The proposed method is illustrated using a simulated example and a case study in a urea packing plant.

KEY WORDS: Experiments; Quality Engineering; Process control; Proportional-Integral control; Minimum mean squared error control.

# 1 Introduction

There are many processes of long duration (e.g., continuous chemical processes) that cannot be made insensitive to the effect of noise by using robust parameter design (or, briefly, parameter design). The use of control is inevitable in these situations. Feedback control involves measurement of the output at regular intervals and compensation for the effect of the uncontrollable disturbance through a controllable process parameter. To understand the combined role of parameter design and feedback control in reducing process variation, consider a simple model

$$Y_t = -2 + 2x - N_t + 0.5xN_t + 2C_{t-1} + z_t, \quad (1)$$

where  $x$  is a control factor that is not changed during production,  $N$  is a noise factor and  $C$  is a control factor that is adjusted to compensate for the unobservable disturbance  $z$ . Changing  $C$  by one unit at time  $t - 1$  produces a 2 units change in  $Y$  at time point  $t$ . We assume that  $N$  and  $z$  are random variables with mean 0 and variance 1. Suppose the target value of  $Y$  is 10. Clearly, if we can set  $x = 4$  and  $C = 2$ , then the target is achieved on average, and we have  $Var(Y) = 2$ . Instead, if we set  $x$  to 2 and  $C$  to 4, then the effect of  $N$  on  $Y$  is removed, and the target is still achieved with a much lower variance of 1.

Now suppose that instead of being a white noise process,  $z_t$  is a non-stationary disturbance which makes the output  $Y$  unstable. In such a case, one can set  $C$  to an initial value  $C_0 = 4$ , and keep on adjusting  $C_t$  with a view to compensate for the disturbance  $z$ , make the process stable and consequently minimize the variation of  $Y$  around the target. In actual practice, this can be achieved by obtaining a forecast of  $z_t$  at time point  $t-1$  from the past observations and adjusting  $C$  based on such a forecast.

Thus, there are two objectives to be fulfilled - one is to find a robust setting for  $x$ , and the other is to find the optimum control law. One way of achieving this may be to fix  $C$  at a certain level and conduct a parameter design experiment to find the optimum setting of  $x$ , and then fix  $x$  at its optimal level and determine the optimal control law. This is referred to as a *two-stage approach*. Such an approach, though not found in the robust design or the control theory literature, is not very difficult to implement. However, such a two-stage

approach for quality improvement may not always work well. For example, if we have a model of the form

$$Y_t = -2 + 2x - N_t + 0.5xN_t + (2 - 0.75x)C_{t-1} + z_t, \quad (2)$$

where  $z_t$  is an autoregressive process of order 1 such that  $z_t = a_t + \phi z_{t-1}$  and  $\text{var}(a_t) = \sigma^2(x) = (1 - 0.5x)^2$ , then obviously the choice of  $x$  would have an impact on the effect of  $N$  on  $Y$  (i.e., robustness of the process) as well as on the control law. Thus the control law would depend on the parameter design solution and vice-versa and a two-stage approach may yield a sub-optimal solution.

Joseph (2003) developed a general parameter design methodology for systems with feed-forward control. In this article, we propose an integrated approach to conduct a parameter design experiment for systems with feedback control. In Section 2, we describe an industrial scenario as a motivating example. In Section 3 we give an overview of some common process inertia models and feedback control schemes. In Section 4, a framework for parameter design with feedback control is proposed for a specific class of process inertia models (pure gain) and the discrete proportional-integral (PI) control scheme. In this Section, we also define the two-stage approach and compare it with the proposed single-stage approach. In Section 5 the proposed methodology is demonstrated through a simulation experiment. In Section 6 we discuss the extension of our proposed framework to minimum mean squared error (MMSE) feedback control scheme. Section 7 illustrates the proposed approach with an example from a packing plant. Section 8 contains concluding remarks and future research directions.

## 2 Motivating Example

As a motivating example we consider the packing experiment described by Dasgupta, Sarkar and Tamankar (2002). The paper describes an automated packing process in which the input material flows into the machine from a hopper. The target weight can be pre-set. There are several control factors  $\mathbf{X}$ , which are set at the beginning of production and usually not altered.

Let  $Y$  denote the response (weight of packed bag) and  $T$  denote the target weight. When a bag is packed, the material flows into the bag in two stages, viz. main (coarse) feed stage (when the material flows into the bag thick and fast) and dribble (fine) feed stage (when the material just trickles down into the bag). In-flight material compensation  $C$  determines how early the main feed will be cut off. The main-feed cut-off value is  $T - (C + \text{Dribble feed quantity})$ . For example, if  $C$  is set to zero, and the target weight is 50 lb, and dribble feed quantity = 12 lb, then the main feed will be cut-off at  $(50-12)=38$  lb. But after the main feed is cut off, there will still be some material flow, which will result in  $Y$  being greater than 50 lb. If  $C$  is now increased to 1 lb, then the main feed will be cut off at  $(50-12-1) = 37$  lb, and  $Y$  will consequently be reduced.  $C$  is therefore used as an on-line adjustment parameter to compensate for the effect of noise. The noise is strong and is a manifestation of a multitude of small effects, none of which can be measured individually. However, an off-line noise factor that can be controlled to some extent for experimental purposes is the material composition (course/fine/lumpy).

Among the set of control factors  $\mathbf{X}$ , some are likely to interact with noise and/or with  $C$ . Further, the variance of the unobservable noise is also expected to depend on some of the control factors. This is thus a case of robust parameter design with feedback control. The actual experiment and analysis of experimental data will be discussed in Section 7.

## 3 Feedback control schemes, models for process inertia and role of DOE

### 3.1 Feedback control schemes and process inertia

Suppose the response  $Y$  has a target  $T$ . Corresponding to time  $t$ , let  $Y_t$  denote the value of  $Y$ ,  $e_t = Y_t - T$  denote the deviation of the response from the target and  $C_t$  denote the value of the adjustment factor. Further, assume that  $Y$  and  $C$  are linked by the following transfer function

$$Y_t = \beta(Y_{t-1}, Y_{t-1}, \dots, C_t, C_{t-1}, \dots) + z_t, \quad (3)$$

where  $z_t$  is the unobservable disturbance. In any feedback control scheme, a correction is given to  $C_t$  on the basis of the observed output error  $e_t$  through a control equation  $C_t = f(e_t, e_{t-1}, \dots)$ .

There is a vast literature on feedback control schemes (e.g., Astrom (1970); Davis and Vinter (1985); Box, Jenkins and Reinsel (1994); Seborg, Edgar and Mellichamp (1989); Del Castillo (2002)). Among various control schemes, the discrete proportional-integral (PI) control schemes have received particular attention because of their simple structure and ease of implementation. In a discrete PI control scheme, the control equation is of the form

$$-C_t = k_0 + k_p e_t + k_I \sum_{i=1}^t e_i, \quad (4)$$

where  $k_p$  and  $k_I$  are positive constants that determine the amount of proportional and integral control. In the example cited in Section 2, the controller is a special case of the discrete PI controller with  $k_p = 0$  (integral control).

Another commonly used feedback control scheme is the minimum mean squared error (MMSE) scheme. Under certain model assumptions and choice of parameters, the discrete PI control scheme and MMSE schemes can be shown to be equivalent (Box and Luceno 1997). However, in general, the PI schemes are seen to be quite efficient over a broad range of the parameter space. Furthermore, as shown by Tsung, Wu and Nair (1998), the PI schemes are more robust to model misspecification than MMSE schemes.

The transfer function in (3) can also be of various types. A simple first-order dynamic model that characterizes many processes of practical interest is given by the following equation

$$Y_t = \alpha + \delta Y_{t-1} + g(1 - \delta)C_{t-1} + z_t, \quad (5)$$

where  $0 < \delta < 1$

A further simplification of (5) can be achieved by assuming that essentially all the change induced by  $C$  will occur in a single time interval. This corresponds to setting  $\delta = 0$  in (5), i.e.,

$$Y_t = \alpha + gC_{t-1} + z_t. \quad (6)$$

This is called the *pure-gain model*. Box & Kramer (1992) considered primarily the pure gain model in their discussion on feedback control.

In Section 4, while developing a framework for robust parameter design with feedback control, we shall restrict attention to the pure-gain dynamics and the integral control scheme.

### 3.2 Choice of control scheme parameters and role of DOE

It is clear that under the discrete PI control scheme, the control can be poor or unstable if the constant  $k_I$  is incorrectly chosen. One way of selection of  $k_I$  is to study the nature of the underlying time series model for  $z_t$  and use this information for optimum selection of  $k$ . For example, if  $z_t$  is an ARIMA(0,1,1) process with parameter  $\lambda$  then under model (5),  $k_p = 0$  and  $k_I = \lambda/g$  result in minimum output variation (Box, Jenkins and Reinsel (1994), Chapter 13).

Suppose a controller has been hooked up to a system and is approximately of right design but is mistuned. One may tune it by formally identifying and fitting models for the process disturbance and dynamics. However, such an approach may be too tedious for routine use. Different experimental approaches for tuning of controller parameters have found place in control theory and chemical engineering literature. These methods were originally based on the trial and error approach (Ziegler and Nichols (1942)), but later, methods based on a single experiment were proposed (e.g., Cohen and Coon (1953), Yuwana and Soberg (1982)). However, it was felt that a sequential approach would be more appropriate for exploring the optimal values of the controller parameters (Carpenter and Sweeny (1965)). As Box and Kramer (1992) point out, to avoid upsetting the system, experimental runs may be made in the evolutionary operation mode, and the response surface methodology may be used to explore and optimize the important factors. Nakano and Jutan (1994) first used the response surface methodology with integral of the squared error (ISE) as the objective function to tune PI controllers. This idea was extended to track dynamic optima by Edwards and Jutan (1997) and Jiang and Jutan (2000).

However, in case a controller has to be set up from scratch, or we have a controller whose basic design is inappropriate, one has to design and conduct a more elaborate experiment

to identify the appropriate models for process disturbance and dynamics. We shall consider both these situations in our proposed framework described in the following Section.

## 4 A framework for robust design with first-order pure-gain dynamic models and discrete PI control scheme

### 4.1 Framework and statistical model

Figure 1 depicts a model for feedback control in the presence of control and noise factors. Let  $\mathbf{X} = (X_1, X_2, \dots, X_p)'$  denote the set of control factors that can only be changed at the process set-up. Let  $\mathbf{R} = (R_1, R_2, \dots, R_q)'$  denote the set of uncontrollable noise factors. Components of  $\mathbf{R}$  interact with the components of  $\mathbf{X}$ . We can write  $\mathbf{R} = \{\mathbf{N}, \mathbf{Z}\}$ , where  $\mathbf{N}$  denotes the set of noise factors that can be deliberately varied during the experiment and  $\mathbf{Z}$  denotes the set of remaining noise factors that cannot be identified or controlled during experimentation. To develop the framework, we assume that all the components of  $\mathbf{N}$  are white noise. We also assume that none of the noise factors are measurable online during production.

In addition, we have a control factor  $C$  that is adjusted *on-line* during production. This control factor, also called *adjustment* factor in robust design literature (Wu and Hamada 2000, Chapter 10), is such that it affects the mean of the response but not its variance. In other words, it does not interact with any of the noise factors.

The response  $Y$  and control factor  $C$  are linked by a transfer function of the form

$$Y_t = \beta(\mathbf{X}, \mathbf{N}, \mathbf{Z}, C_{t-1}, C_{t-2}, \dots),$$

which can be rewritten as

$$Y_t = \beta(\mathbf{X}, \mathbf{N}, C_{t-1}, C_{t-2}, \dots) + z_t,$$

where  $\{z_t\}$  is the disturbance due to unobservable and uncontrollable noise factors  $\mathbf{Z}$  and may be stationary or non-stationary. Since  $\mathbf{Z}$  interacts with  $\mathbf{X}$  but not with  $C$  (by definition of adjustment factor), the variance of  $z_t$  may be assumed to depend on  $\mathbf{X}$ , but not on  $C$ .

At time  $t$ , a correction is given to  $C_t$  on the basis of the observed output error  $e_t$  through a control equation  $C_t = f(e_t, e_{t-1}, \dots)$ . As discussed in Section 3.1, we shall consider the following forms for the functions  $\beta$  and  $f(e_t, e_{t-1}, \dots)$ .

$$\begin{aligned}\beta(\mathbf{X}, \mathbf{N}, C_{t-1}, C_{t-2}, \dots) &= \alpha_0(\mathbf{X}, \mathbf{N}) + g(\mathbf{X})C_{t-1}, \\ f(e_t, e_{t-1}, \dots) &= -k_0 - k_I \sum_{i=1}^t e_i.\end{aligned}$$

We thus postulate the following first-order pure-gain dynamic model

$$Y_t = \alpha_0(\mathbf{X}, \mathbf{N}) + g(\mathbf{X})C_{t-1} + z_t. \quad (7)$$

If  $T$  denotes the target, and  $e_t = Y_t - T$  denotes the deviation from the target, then,

$$e_t = \alpha(\mathbf{X}, \mathbf{N}) + g(\mathbf{X})C_{t-1} + z_t, \quad (8)$$

where  $\alpha(\mathbf{X}, \mathbf{N}) = \alpha_0(\mathbf{X}, \mathbf{N}) - T$ .

Note that this is essentially the same as (6), the added aspect being the dependence of the dynamics on  $\mathbf{X}$  and  $\mathbf{N}$ . It is thus imperative that, if an integral control scheme is employed for such a process,  $k_I$  necessarily has to be a function of  $\mathbf{X}$  and  $\mathbf{X}$  has to be such that the output is least sensitive to the effect of  $\mathbf{N}$ .

## 4.2 Performance measure and its optimization

Let us assume that  $z_t = \sum_{j=0}^{\infty} \psi_j a_{t-j}$ , where  $\{a_t\}$  is a white noise process with zero mean and variance  $\sigma^2(\mathbf{X})$ . Recalling that in an integral control scheme  $C_{t-1}$  is set to  $-k_0 - k_I \sum_{i=1}^{t-1} e_i$ , we have from (8),

$$\begin{aligned}e_t &= \alpha(\mathbf{X}, \mathbf{N}) - g(\mathbf{X}) \left( k_0 + k_I \sum_{i=1}^{t-1} e_i \right) + z_t \\ &= \left( \alpha(\mathbf{X}, \mathbf{N}) - g(\mathbf{X})k_0 \right) - g(\mathbf{X})k_I \sum_{i=1}^{t-1} e_i + \sum_{j=0}^{\infty} \psi_j a_{t-j}.\end{aligned} \quad (9)$$

Clearly, the objective is to select  $\mathbf{X}$  and the control law in such a way that the variance of  $e_t$  is minimized. For the type of processes considered here, the duration of the control session

will be long. Thus, it is reasonable to consider  $PM(\mathbf{X}, k_I) = Var(e_t)$  as the appropriate performance measure, provided the output  $e_t$  is asymptotically stable with mean zero.

We have

$$\begin{aligned} Var(e_t) &= Var_a E_{\mathbf{N}}(e_t|a) + E_a Var_{\mathbf{N}}(e_t|a) \\ &= Var(u_t) + \pi(\mathbf{X}), \end{aligned} \quad (10)$$

where

$$\begin{aligned} u_t &= E_{\mathbf{N}}(e_t|a) \\ &= \left( \alpha(\mathbf{X}) - g(\mathbf{X})k_0 \right) - g(\mathbf{X})k_I \sum_{i=1}^{t-1} u_i + \sum_{j=0}^{\infty} \psi_j a_{t-j} \end{aligned}$$

and  $\alpha(\mathbf{X}) = E_{\mathbf{N}}(\alpha(\mathbf{X}, \mathbf{N}))$ ,  $\pi(\mathbf{X}) = Var_{\mathbf{N}}(\alpha(\mathbf{X}, \mathbf{N}))$ .

It is seen that if model (8) and the distribution of  $\mathbf{N}$  is known, then we can compute the performance measure  $PM(\mathbf{X}, k_I)$  by substituting  $Var(u_t)$  in (10). For a given process disturbance model (i.e., given the weights  $\psi_j$ )  $Var(u_t)$  can be obtained through routine but tedious derivations (Tsung, Wu and Nair, 1996).  $Var(u_t)$  will be a function of  $\mathbf{X}$  and  $k_I$ , and will be of the form  $V(\mathbf{X}, k_I)\sigma^2(\mathbf{X})$ . Clearly,  $V(\mathbf{X}, k_I)$  depends on  $\mathbf{X}$  through the function  $g(\mathbf{X})$ . For example, if  $z_t$  is an ARIMA(0,1,1) process with parameter  $\lambda$ , one can obtain the following expression for  $V(\mathbf{X}, k_I)$  by slightly extending the result of Box and Kramer (1992),

$$\begin{aligned} V(\mathbf{X}, k_I) &= \frac{1 + \theta^2 - 2\phi(\mathbf{X}, k_I)\theta}{1 - \phi^2(\mathbf{X}, k_I)}, \quad \text{if } -1 < \phi(\mathbf{X}, k_I) < 1, \\ &= \infty \quad \text{otherwise,} \end{aligned} \quad (11)$$

where  $\phi(\mathbf{X}, k_I) = 1 - g(\mathbf{X})k_I$  and  $\theta = 1 - \lambda$ .

We thus have

$$PM(\mathbf{X}, k_I) = V(\mathbf{X}, k_I)\sigma^2(\mathbf{X}) + \pi(\mathbf{X}). \quad (12)$$

Stability of the output  $e_t$  can be ensured by choosing  $k_I$  within appropriate stability region  $\kappa(\mathbf{X})$ , which depends on the nature of the disturbance  $z_t$ .

From (8), we can write for  $t = 1$ ,

$$e_1 = \alpha(\mathbf{X}, \mathbf{N}) + g(\mathbf{X})C_0 + z_1.$$

Noting that the starting value of  $C_0$  is  $k_0$ , we have

$$E(e_1) = \alpha(\mathbf{X}) + g(\mathbf{X})k_0.$$

Thus, to ensure that the output is asymptotically stable around zero, we must have  $k_0 = -\alpha(\mathbf{X})/g(\mathbf{X})$  and  $k_I \in \kappa(\mathbf{X})$ .

Next, the following *two-step optimization* is performed :

1. Minimize  $PM(\mathbf{X}, k_I)$  subject to  $\mathbf{X} \in [\mathbf{X}^L, \mathbf{X}^U]$  and  $k_I \in \kappa(\mathbf{X})$ , where  $[\mathbf{X}^L, \mathbf{X}^U]$  denotes the experimental range for the control variables. Let  $\mathbf{X}^*$  and  $k_I^*$  denote optimum  $\mathbf{X}$  and  $k_I$  respectively.
2. Obtain the optimal  $k_0$  as  $k_0^* = -\alpha(\mathbf{X}^*)/g(\mathbf{X}^*)$ .

To illustrate the computation of the performance measure, let us consider the following example :

$$e_t = -2x - N + 0.5xN + (2 - 0.75x)C_{t-1} + z_t.$$

We assume that  $z_t = \phi z_{t-1} + a_t$  is an AR(1) process with  $\phi = 0.7$ ,  $E(a_t) = 0$ ,  $Var(a_t) = (2 - 0.8x)^2$  and  $N$  is a random variable with mean 0 and variance 1.

Extending the results of Tsung, Wu and Nair (1996) to the current model, it can easily be seen that the stability region here is  $\kappa(\mathbf{X}) = \{k_I : |1 - g(\mathbf{X})| \leq 1\}$ . Also, an appropriate expression for  $V(\mathbf{X}, k_I)$  under this model is given by

$$\begin{aligned} V(\mathbf{X}, k_I) &= \frac{2}{(1 + \eta(\mathbf{X}))(1 - \phi\eta(\mathbf{X}))(1 + \phi)}, \quad \text{where} \\ \eta(\mathbf{X}) &= 1 - g(\mathbf{X})k_I. \end{aligned} \tag{13}$$

Substituting appropriate expressions in (13) and (12), we get

$$PM(\mathbf{X}, k_I) = \frac{1.1765}{\left(1 + \{1 - k_I(2 - 0.75x)\}\right) \left(1 - 0.7\{1 - k_I(2 - 0.75x)\}\right)} (2 - 0.8x)^2 + (1 - 0.5x)^2.$$

If the experimental region is  $0 \leq x \leq 5$ , the problem is to

$$\begin{aligned} & \text{minimize } PM(\mathbf{X}, k_I) \text{ subject to} \\ & 0 \leq x \leq 5, \\ & k_I \in \{k_I : |1 - k_I(2 - 0.75x)| < 1\}. \end{aligned}$$

Using a simple non-linear constrained optimization function from MATLAB, we find that the minimum is obtained at  $x^* = 2.2990$ ,  $k_I^* = 2.8497$  and the corresponding value of the performance measure is  $PM(x^*, k_I^*) = 0.0374$ . Assuming  $E(N) = 0$  and  $var(N) = 1$ , and substituting  $x^* = 2.299$ , we obtain the optimal  $k_0$  as  $k_0^* = -\alpha(\mathbf{X}^*)/g(\mathbf{X}^*) = 16.6745$ .

The performance of the system without control may be evaluated by substituting  $k_I = 0$  in the expression for performance measure, which gives  $PM(\mathbf{X}, 0) = \frac{(2-0.8x)^2}{(1-\phi^2)} + (1 - 0.5x)^2$ . Here  $PM(\mathbf{X}^*, 0) = 0.0730 > PM(\mathbf{X}^*, k_I^*)$ , demonstrating that the control will be effective.

### 4.3 Comparison with a two-stage approach

The interesting aspect of the proposed approach is that, the optimal settings of the control variables  $\mathbf{X}$  and the controller parameters  $k_0$  and  $k_I$  are obtained by conducting a single experiment. Let us now compare this *single-stage* approach with a two-stage approach where at the first stage robust level of  $\mathbf{X}$  is decided by conducting a traditional parameter design experiment and next the optimal control law is obtained.

From (8), we have,

$$\begin{aligned} e_1 &= \alpha(\mathbf{X}, \mathbf{N}) + g(\mathbf{X})C_0 + z_1 \\ &= \alpha(\mathbf{X}, \mathbf{N}) + g(\mathbf{X})C_0 + a_1, \quad [\text{assuming } \psi_0 = 1.] \end{aligned}$$

Setting  $C_0 = C_0^*$ , one can perform a traditional parameter design experiment to find out the optimum setting of  $\mathbf{X}$  by minimizing  $Var(e_1) = \pi(\mathbf{X}) + \sigma^2(\mathbf{X})$ . Here, unlike traditional robust parameter design, the second-step of optimization (adjusting the mean to target by proper choice of  $\mathbf{X}$ ) will not be done, since  $C$  is considered to be the only adjustment factor here. Let  $\tilde{\mathbf{X}}$  denote the optimum  $\mathbf{X}$  obtained this way.

The second stage of the optimization consists of obtaining the optimal  $k_I$  for the chosen

setting  $\mathbf{X} = \tilde{\mathbf{X}}$ . This is equivalent to obtaining

$$\tilde{k}_I = \arg \min_{k_I \in \kappa(\tilde{\mathbf{X}}), \mathbf{X} = \tilde{\mathbf{X}}} \text{Var}(e_t) = \arg \min_{k_I \in \kappa(\tilde{\mathbf{X}})} V(\tilde{\mathbf{X}}, k_I).$$

If  $\mathbf{X}^*$  and  $k_I^*$  denote the optimum  $\mathbf{X}$  and  $k_I$  obtained using the proposed single stage approach, then

$$\begin{aligned} PM(\mathbf{X}^*, k_I^*) &= \min_{\mathbf{X} \in (X^L, X^U), k_I \in \kappa(\mathbf{X})} V(\mathbf{X}, k_I) \sigma^2(\mathbf{X}) + \pi(\mathbf{X}) \\ &\leq PM(\tilde{\mathbf{X}}, \tilde{k}_I). \end{aligned}$$

Thus, in general, the asymptotic variance of the output obtained using the proposed single-stage approach will be less than or equal to that for the two-stage approach. To demonstrate this numerically, consider the example in Section 4.2. Here, the two-stage approach would consist of the following :

- Stage I :

$$\begin{aligned} \tilde{x} &= \arg \min_{0 < x < 5} \left( \sigma^2(x) + \pi(x) \right) \\ &= \arg \min_{0 < x < 5} \left( (1 - 0.5x)^2 + (2 - 0.8x)^2 \right) = 2.3596. \end{aligned}$$

- Stage II

$$\tilde{k}_I = \arg \min_{k_I \in \kappa(\tilde{x})} V(\tilde{x}, k_I) = 3.4117.$$

The proposed single-stage approach would yield the following result :

$$(x^*, k_I^*) = \arg \min_{x \in (0, 5), k_I \in \kappa(x)} V(x, k_I) \sigma^2(x) + \pi(x) = (2.2990, 2.8497).$$

We have,  $0.0397 = PM(\tilde{x}, \tilde{k}_I) > PM(x^*, k_I^*) = 0.0374$ , which indicates that the two-stage approach yields a sub-optimal solution.

However, it may be noted that if  $\min_{k_I \in \kappa(\mathbf{X})} V(\mathbf{X}, k_I) = 1$  for all  $\mathbf{X}$ , then the two procedures would lead to identical results. Physically this means that through the control

mechanism, it is possible to reduce the variation of the output to the variance of the underlying white noise. This will hold good for the ARIMA(0,1,1) model, where by choosing  $k_I = \frac{\lambda}{g(\mathbf{X})}$ ,  $V(\mathbf{X}, k_I)$  can be reduced to 1.

Another drawback of the two-stage approach is the improper selection of  $k_0$ , since it will be impossible to distinguish  $\alpha(\mathbf{X}, \mathbf{N})$  from  $g(\mathbf{X})$  in the first-stage model obtained by keeping  $C$  fixed. This will give rise to instability in the process during the initial phase of production.

## 4.4 Design of experiments and analysis of data

In order to express  $PM$  as a suitable function of  $\mathbf{X}$  and  $k_I$ , one may conduct a suitable open-loop experiment with  $\mathbf{X}, \mathbf{N}$  and  $C$  as experimental factors to estimate model (8), fit an appropriate time series model for  $z_t$  and then obtain an expression for  $PM$  by considering an appropriate distribution of  $\mathbf{N}$ . This approach is known as *response modeling* (Wu and Hamada 2000, Chaps 10 and 11). An alternative procedure is to directly model  $PM$  as a function of  $\mathbf{X}$  and  $k_I$  by treating  $k_I$  as an experimental factor and conducting the experiment with the control loop. This is called *performance measure modeling*. In the following two subsections, we discuss the design and analysis of experiments under these two approaches.

### 4.4.1 Response modeling approach

Recall that in Section 4.2, we had assumed the dependence of  $\sigma_a^2$  (the variance of  $a_t$ ) on  $\mathbf{X}$ . Thus, the response modeling may be thought of as a two-step approach:

1. Fitting a transfer function of the form  $e_t = \alpha + gC_{t-1} + z_t$  for various combinations of  $\mathbf{X}$  and  $\mathbf{N}$ .
2. Modelling  $\alpha$  as a function of  $(\mathbf{X}, \mathbf{N})$ ;  $g$  and  $\sigma_a^2$  as functions of  $\mathbf{X}$ .

To achieve this objective, we may use a cross array design between  $\mathbf{X}$  and  $\mathbf{N}$  and nest all the levels of  $C$  within each  $\mathbf{X}, \mathbf{N}$  combination. Thus, for each combination of  $\mathbf{X}$  and  $\mathbf{N}$ , a time series in  $Y_t$  and hence  $e_t$  will be obtained by changing the levels of  $C$ . As in the simulated experiment described on p. 442 of Box et al. (1994), each level of  $C$  may be held constant

for a fixed time, and  $\tau$  observations may be generated. Instead of employing a cross array design, we may use a single array as well, ensuring that all the interactions between  $\mathbf{X}$  and  $\mathbf{N}$  are estimable. Details of cross array and optimal single array designs may be found in Chapter 10 of Wu and Hamada (2000).

As indicated above, the analysis consists of the following two broad stages:

1. For each combination of  $\mathbf{X}$  and  $\mathbf{N}$ ,
  - (a) Identification of the form of the disturbance  $z_t = \sum_{j=0}^{\infty} \psi_j a_{t-j}$  using autocorrelation function (ACF) plots and partial autocorrelation function (PACF) plots;
  - (b) Estimating the parameters ( $\alpha$ ,  $g$ ,  $\psi_j$ 's,  $\sigma_a^2$ ) using a constrained iterative nonlinear least squares algorithm (Box et al. 1994, Chapter 7).
2. Treating  $\alpha$ ,  $g$  and  $\sigma_a^2$  as three different responses, we identify significant control factors and control $\times$ noise interactions and fit  $\hat{\alpha} = \alpha(\mathbf{X}, \mathbf{N})$ ,  $\hat{g} = g(\mathbf{X})$ ,  $\hat{\sigma}_a^2 = \sigma_a^2(\mathbf{X})$ .

Note that if we assume that  $\sigma_a^2$  does not depend on  $\mathbf{X}$ , i.e., all the parameters associated with the disturbance  $z_t$  are free of  $\mathbf{X}$ , then the experiment can be considerably simplified. In such a case we may estimate the time series parameters from a single experimental run for fixed  $\mathbf{X} = \mathbf{X}^*$  and  $\mathbf{N} = \mathbf{N}^*$ . Next, we may conduct a cross array design  $D(\mathbf{X}) \otimes D(\mathbf{N}) \otimes D(C)$  and estimate model (8) directly from the experimental data.

Although the response modeling approach provides an in-depth understanding of the underlying phenomena, it is clear that there is a possibility that the experiment will be very large and will involve intensive computation. Further, it is obvious, that this experiment has to be run with an open loop. For systems in which controllers have already been installed, industrial personnel would usually be reluctant to run open-loop experiments. Thus, when the objective is to achieve robustness of a system that already has a feedback controller, the performance measure modeling approach discussed in the following section will be appropriate.

#### 4.4.2 Performance measure modeling

Since this approach can be thought of as modeling the performance measure as a function of  $\mathbf{X}$  and  $k_I$ , it would be reasonable to use a cross array design between  $\mathbf{X}$  and  $k_I$ . The various noise combinations may be nested within each  $(\mathbf{X}, k_I)$  combination. Thus, for each combination of  $\mathbf{X}$  and  $k_I$ , a time series in  $Y_t$  and hence  $e_t$  will be obtained by changing the levels of  $\mathbf{N}$ . In order to reduce the run size,  $\mathbf{X}$  and  $k_I$  can be accommodated in a single array. In this case, one must ensure that all or most of the interactions between  $k_I$  and  $\mathbf{X}$  are estimable.

Note that, in this set-up, the adjustment factor  $C$  need not be included in the experiment as it will be automatically changed during the course of this closed loop experiment. However,  $k_0$  corresponds to the initial value of  $C$  (at time 0) and may be treated as 'another' control factor.

Selection of levels for  $k_I$  is a very important aspect, regarding which the experimenter has to be careful. At least three levels should be chosen for  $k_I$  since for any given setting, the variation in the output is approximately a quadratic function of  $k_I$  and the quadratic effect of  $k_I$  should be important. However, keeping in mind the fact that grossly improper choice of  $k_I$  may make the output unstable and upset the entire process, some amount of caution would usually be exercised in the selection of its levels. Thus, it may not be possible to hit the optimum with a single experiment and additional runs may be added later in the evolutionary operation mode as suggested by Box and Kramer (1992).

Let  $Y_{ijkt}$  be the  $t^{\text{th}}$  measured value of the characteristic at the  $i^{\text{th}}$  level of  $\mathbf{X}$ ,  $j^{\text{th}}$  level of  $k_I$ , and  $k^{\text{th}}$  level of  $\mathbf{N}$  ( $i = 1, 2, \dots, I$ ,  $j = 1, 2, \dots, J$ ,  $k = 1, 2, \dots, K$ ,  $t = 1, 2, \dots, \tau$ ). Let  $e_{ijkt} = Y_{ijkt} - T$ . Then we compute the estimated value of the performance measure corresponding to the  $i^{\text{th}}$  level of  $\mathbf{X}$  and  $j^{\text{th}}$  level of  $k_I$  as

$$\widehat{PM}_{ij} = \frac{1}{K\tau - 1} \sum_{t=1}^{K\tau} \left( e_{ijkt} - \overline{e_{ij.}} \right)^2,$$

where

$$\overline{e_{ij.}} = \frac{1}{K\tau} \sum_{k=1}^K \sum_{t=1}^{\tau} e_{ijkt}.$$

Next, fit the linear regression model

$$\ln \widehat{PM} = f(\mathbf{X}, k_I), \quad (14)$$

and we determine optimum values of  $\mathbf{X}$  and  $k_I$  by optimizing the fitted function  $f(\mathbf{X}, k_I)$ .

The above analysis is based on the assumption that all the factor-level combinations would produce a stationary output around  $T$  thereby ensuring the finiteness of  $V(e_t)$ . As seen in Section 4.2, this may not be the case if  $k_I$  is chosen at a level beyond the range within which it is capable of producing a stationary output. If it is found that for some level combinations  $(i, j, k)$ ,  $\overline{e_{ij..}}$  is largely different from zero, it would be pragmatic to use the sample mean squared error  $m_{ij}^2 = \frac{1}{K\tau-1} \sum_{t=1}^{K\tau} (Y_{ijkt} - T)^2$  instead of  $\widehat{PM}_{ij}$ .

## 5 A simulation study

Let us revisit the example discussed in Section 4.2 for a simulation study:

$$e_t = -2X_1 - N + 0.5X_1N + (2 - 0.75X_1)C_{t-1} + z_t.$$

We assume that  $z_t = \phi z_{t-1} + a_t$  is an AR(1) process with  $\phi = 0.7$ ,  $E(a_t) = 0$ ,  $Var(a_t) = (2 - 0.8X_1)^2$  and  $N$  is a random variable with mean 0 and variance 1. Also assume that besides  $X_1$ , there is another control factor  $X_2$ , which the experimenter will consider for experimentation.

The experimenter's objective is to choose robust settings of  $X_1, X_2$  and the optimal integral control law  $C_t = -k_0 - k_I \sum_{i=1}^t e_i$  so that the deviation of  $e$  is minimal around zero.

### 5.1 Response modeling

Three levels for each of the two control factors  $X_1$  and  $X_2$ , two levels for the noise factor  $N$  and three levels for the adjustment factor  $C$ . The levels of the factors are shown in Table 1. The control array is a  $3^2$  design, and each of the 9 combinations of the control factors are crossed with the two levels of the noise factor  $N$ . For each of the 18 control-noise combinations, each level of  $C$  is held constant till 10 observations are generated. From these

30 observations in each control-noise cell, a transfer function of the form  $e_t = \alpha + gC_{t-1} + z_t$  is fit using weighted least squares, and an appropriate time series model is fit to the residuals. The summarized experimental data are shown in Table 2.

It is found that the residuals constitute an AR(1) process with estimated parameter  $\hat{\phi} = 0.61$ . As described in Section 4.4, treating  $\alpha, g$  and  $\sigma^2$  as three different responses, using the methodology described in Wu and Hamada (2000), it is found that the significant factors affecting  $\alpha$  are  $X_1$  and  $N$ , and  $g$  and  $\sigma^2$  are affected by  $X_1$  only.

The following models are obtained:

$$\begin{aligned}\alpha(\mathbf{X}, \mathbf{N}) &= -1.99X_1 - 1.061N + 0.527X_1N, \\ g(\mathbf{X}) &= 2.02 - 0.76X_1, \\ \sigma^2(\mathbf{X}) &= 4.36 - 3.51x + 0.71X_1^2.\end{aligned}$$

Consequently, the complete fitted model is

$$e_t = -1.99X_1 - 1.061N + 0.527X_1N + (2.02 - 0.76X_1)C_{t-1} + z_t,$$

where  $z_t = \phi z_{t-1} + a_t$  is an AR(1) process with  $\phi = 0.61$ , and  $Var(a_t) = 4.36 - 3.51x + 0.71X_1^2$ .

Substituting all the estimated parameters of the model in (13) and (12), we get

$$PM(\mathbf{X}, k_I) = \frac{1.2422}{\left(1 + \{1 - k_I(2.02 - 0.76X_1)\}\right) \left(1 - 0.61\{1 - k_I(2.02 - 0.76X_1)\}\right)} (4.36 - 3.51X_1 + 0.71X_1^2) + (1.061 - 0.527X_1)^2.$$

Minimizing the above function subject to the constraints  $2.2 \leq X_1 \leq 2.4$  and  $0 \leq k_I \leq \frac{2}{2.02 - 0.76X_1}$ , we get  $X_1^* = 2.3569$  and  $k_I^* = 2.9739$ . These are fairly good estimates of the true optimal values  $(X_1^*, k_I^*) = (2.2990, 2.8497)$ . The optimal value of  $k_0$  is obtained as  $k_0^* = 20.5$ .

## 5.2 Performance measure modeling

For this experiment, we choose three levels for each of the factors  $X_1, X_2, k_0, k_I$  and two levels for the noise factor  $N$ . The levels of the control factors  $X_1, X_2$ , noise factor  $N$  and the control equation parameters  $k_0$  and  $k_I$  are shown in Table 3. A  $3^4$  design is used, in which,

for each combination of  $X_1, X_2, k_0, k_I$ , 100 observations are generated with  $N$  at level  $-1$  and 100 observations with  $N$  at level  $+1$ . The mean squared error of these 200 observations are computed. The experimental data, i.e., the mean squared errors corresponding to the 81 runs are shown in Table 4. Finding out the significant linear and quadratic effects following the methodology described in Chapter 5 of Wu and Hamada (2000) and fitting a second-order regression equation relating  $\log(\text{MSE})$  to the significant variables, the following performance measure model is obtained :

$$\begin{aligned} \log(\widehat{PM}) &= 97.81 - 104.3X_1 + 27.98X_1^2 + 2.047k_0 + 0.00576k_0^2 \\ &+ 5.489k_I + 0.7828k_I^2 - 0.9895X_1k_0 - 4.447X_1k_I. \end{aligned}$$

The problem is thus

$$\begin{aligned} &\text{minimize } \log(\widehat{PM}) \\ &\text{subject to} \\ &2.2 \leq X_1 \leq 2.4, 12 \leq k_0 \leq 24, 1.5 \leq k_I \leq 4.5. \end{aligned}$$

The minimum is obtained at  $X_1 = 2.32, k_0 = 12, k_I = 3.2$ ; which is reasonably close to the true optimum, considering the fact that in this approach we are trying to approximate a complex response surface by a simple second-order polynomial function. A sequential approach may be adopted to explore the response surface and the settings may be fine tuned.

With this simple model involving one control variable, the response modeling and performance measure modeling approach give almost identical results (except for  $k_0$ , which is, of course, not as important as  $\mathbf{X}$  and  $k_I$ ). However, as the underlying model becomes increasingly complex with addition of more control variables, the simple second-order polynomial function obtained from the response modeling method may not be good enough to model the performance measure properly. In such cases, it may be better to conduct a response modeling experiment at the first stage to get a fairly good idea about the significant variables, their interactions with noise and then use a closed-loop performance measure modeling experiment to fine-tune the settings.

## 6 Robust design with the MMSE control scheme and the pure-gain model

Once again, let us consider model (8) with the same assumptions for the process disturbance  $z_t$  as mentioned at the beginning of Section 4.2. Under the MMSE control scheme,  $C_{t-1}$  is to be set in such a way so that the one-step ahead forecast of  $Y_t$  is equal to the target  $T$  (or equivalently, the forecast of  $e_t$  is zero). An elaborate discussion can be found in Box and Luceno (1997).

Thus, under the MMSE scheme for the pure gain model,  $C_{t-1}$  should be such that

$$\hat{e}_{t-1}(1) = \alpha(\mathbf{X}, \mathbf{N}) + g(\mathbf{X})C_{t-1} + \hat{z}_{t-1}(1) = 0. \quad (15)$$

Since  $\alpha(\mathbf{X}, \mathbf{N})$  will not be known in reality, the MMSE control equation can be obtained from the above by taking the expectation over  $\mathbf{N}$ , i.e.,

$$C_{t-1} = \frac{1}{g(\mathbf{X})} \left( -\alpha(\mathbf{X}) - \hat{z}_{t-1}(1) \right). \quad (16)$$

Substituting (14) into (8), we get

$$e_t = \left( \alpha(\mathbf{X}, \mathbf{N}) - \alpha(\mathbf{X}) \right) + (z_t - \hat{z}_{t-1}(1)). \quad (17)$$

Thus, we have

$$\begin{aligned} Var(e_t) &= Var_a E_{\mathbf{N}}(e_t|a) + E_a Var_{\mathbf{N}}(e_t|a) \\ &= Var \left( z_t - \hat{z}_{t-1}(1) \right) + \pi(\mathbf{X}) \\ &= \sigma^2(\mathbf{X}) + \pi(\mathbf{X}), \end{aligned} \quad (18)$$

where  $\pi(\mathbf{X})$  is defined as before.

The last step of (18) follows from the properties of the MMSE forecast (Box et al. 1994, Chapter 5). The fact that under the ARIMA(0,1,1) disturbance, the discrete PI scheme with  $k_I = \frac{\lambda}{g(\mathbf{X})}$  is the same as the MMSE scheme can be easily seen by substituting  $k_I = \frac{\lambda}{g(\mathbf{X})}$  in (11) and observing that  $Var(e_t)$  reduces to the form given by (18).

It is evident that with the MMSE control scheme, only the response modeling approach would work, since it is not possible to establish the control equation and observe the output error under control without estimating model (8). Since MMSE control schemes reduce the variance of the output error to the variance of the underlying white noise, by the argument in Section 4.2, one may use the two-stage approach in this case. However, as discussed before, a two-stage approach will lead to improper choice of the component  $\alpha(\mathbf{X})$  in the control equation.

## 7 A case study

In Section 2 a study on optimization of a control scheme of the packing process of a urea manufacturing plant (Dasgupta et al., 2002) was mentioned as a motivating example. The underlying control scheme was a discrete PI scheme, slightly different from the classical one (see Appendix of Dasgupta et al. 2002). Among the 16 factors listed in the original case study,  $D$  (auto compensation proportional constant) and  $K$  (in-flight material compensation - start) correspond to  $k_I$  and  $k_0$  respectively. The recoded factors and levels of the experiment are given in Table 5. The packing system is shown in Figure 2.

The original experiment was conducted and analyzed somewhat superficially just like “another” robust design exercise, not recognizing the specific roles of factors  $k_I$  and  $k_0$ . It may also be noted that the original experiment does not explicitly consider any noise factor. However, since we have three replicates, we can consider them to correspond to three levels of the noise factor material composition  $N$ .

An  $L_{32}$  orthogonal array (OA) with an idle column was used to design the experiment. The idle column method is a technique to generate three level columns by collapsing two columns in a two-level orthogonal array (Taguchi 1987; Grove & Davis 1991; Huwang, Wu & Yen 2002). Besides the 16 main effects (19 degrees of freedom), provisions were kept for estimation of five interaction effects, viz.  $X_1 \times X_2$ ,  $X_6 \times X_{11}$ ,  $X_6 \times X_9$ ,  $X_7 \times X_{10}$  and  $X_2 \times k_I$ .

Sixty bags were filled for each of the  $32 \times 3 = 96$  level combinations. The target weight was set at  $T = 50.5$  kg.

As mentioned in Section 2, in-flight material composition is the adjustment factor  $C$  in this case.  $\mathbf{X} = \{X_1, \dots, X_{14}\}$  denotes the set of control factors. Note that we can use equation (7) to model  $Y_t$ , the weight of a packed bag at time  $t$ , where  $z_t$  in (7) is the disturbance due to other uncontrollable noise factors such as temperature of the flowing material, moisture content in the hopper, etc. and has a certain time series structure.

The response modelling approach cannot be employed because it would require an open-loop experiment as discussed in Section 4.4.1. Here, since the experiment was conducted with a closed loop with  $k_0$  (starting value of the adjustment factor  $C$ ) and  $k_I$  as two of the experimental factors, the performance measure modelling approach needs to be used. Recall also that in this approach it is not necessary to fit a time series model to  $z_t$ ; rather, one has to express the performance measure (variance or mean squared error of data collected over time) as a function of the control factors,  $k_0$  and  $k_I$ .

Note that in performance measure modelling, for each experimental combination we would generate a time series over the different levels of noise. Such was, however, not the case in the original experiment. For purpose of illustration, we assume that the 180 observations corresponding to each of the 32 experimental runs constitute a single time series. The sample standard deviation of the 180 observations corresponding to each run is shown in Table 6.

Using half-normal plots for the 27 treatment effects (Figure 3), we find that four effects viz.  $X_9X_{6q}$ ,  $k_0$ ,  $X_{14q}$  and  $X_{6q}$  stand apart from the rest. Using Lenth's method (Wu and Hamada 2000, Chapter 4) as a formal test of significance it is seen that all of these 4 effects are significant at 1% level. Note that for any three-level factor  $X$ , we denote the linear and quadratic effects by  $X_l$  and  $X_q$  respectively (Wu and Hamada 2000, Chapter 5).

We notice that neither the main effect of  $k_I$  nor the  $X_2 \times k_I$  interaction (which is estimable) is significant. Another factor that is suspected to interact with  $k_I$  is  $X_3$ , and we explore the possibility of incorporating the  $X_3 \times k_I$  interaction into our analysis by replacing some insignificant effects in the preliminary model. Using the table of interactions for the  $L_{32}$  OA, we find that the  $X_3 \times k_I$  interaction can be estimated from columns 8 and 9 of the OA. Out of these two columns, 8 corresponds to factor  $X_{11}$  which is seen to be insignificant and 9 is a free column to which no other main effect or significant interaction is assigned. Thus we

re-perform the analysis by including the  $X_3 \times k_I$  interaction (with two degrees of freedom) instead of  $X_{11}$ .

The half-normal plot (Figure 4) now identifies 6 effects standing above the rest. Apart from the four that were already seen to be significant, the interaction effects  $X_3k_{I_l}$  and  $X_3k_{I_q}$  turn out to be significant at 1% level.

From the plots of significant main effects (Figure 5) and the  $X_6 \times X_9$  interaction (Figure 6), we choose the optimal settings of  $k_0$ ,  $X_9$ ,  $X_6$  and  $X_{14}$  as  $k_0 = -1, X_9 = 1, X_6 = 1$  and  $X_{14} = 2$ . Note that all of these factors (or interactions involving them) were found significant in the original analysis and the same levels (although different notations were used) were selected as the optimum ones.

The interaction plot of  $X_3 \times k_I$  (Figure 6) suggests that at neither of the two levels of  $X_3$ , the optimum  $k_I$  could be reached. Corresponding to  $X_3 = 1$ ,  $k_I^*(X_3) \leq 0$  while corresponding to  $X_3 = -1$ ,  $k_I^*(X_3) \geq 2$ . We also note that the curve corresponding to  $X_3 = 1$  is slightly convex as expected, whereas that corresponding to  $X_3 = -1$  exhibits a slight concavity, which can be attributed to sampling error or effect of higher order interactions. Since this difference in convexity results in significance of the quadratic component of the interaction, we may only consider the linear component of the interaction while modeling  $\ln(\widehat{PM})$ . From the interaction plot, we choose  $X_3 = -1$  and  $k_I = 2$  as the optimal settings.

The following model is thus obtained:

$$\begin{aligned} \widehat{PM} = & -5.7438 + 1.2408x_{k_0} - 0.832x_6 + 0.3344x_6^2 + 1.0725x_{14} - 0.5863x_{14}^2 \\ & + 0.2362x_3x_{k_I} - 0.1240x_9x_6^2. \end{aligned} \quad (19)$$

Substituting the optimal settings of the control factors in the above model, we get the optimal value of the performance measure as  $\widehat{PM}^* = -8.2788$ , which corresponds to a standard deviation of 0.0159. The original experiment was able to reduce the output standard deviation drastically to 0.031 from the existing value of 0.121. We find from this re-analysis that with the newly recommended settings, it might have been possible to reduce the variation to almost 50% of what had been achieved.

This analysis also points out the importance of adopting a sequential approach for the

performance modeling experiment. It is clear that with wider choices of levels of  $k_I$ , it might have been possible to reduce the output variance even further.

## 8 Concluding remarks

In this article we have developed a framework for robust parameter design of systems with feedback control. The suggested approach can be used to obtain the optimal control law and robust parameter design solution at a single stage. Appropriate performance measures have been developed and the design and analysis of experiments for estimation and optimization of these performance measures have been discussed. The advantages of using the proposed approach as compared to a two-stage approach have been discussed. The benefits of using the proposed method have been demonstrated using a simulation study and an example from a packing plant.

Although we have considered the pure-gain dynamic model and primarily the discrete integral control scheme, it should be possible to extend the proposed methodology to a much more generic class of models and other control schemes.

One important assumption in the proposed framework is that the noise factors (that appear in the experiment) are uncorrelated over time. There are many examples of noise factors, e.g., material composition in the case study, which are uncorrelated or weakly correlated over time and may be well approximated by a white noise disturbance. However, in dynamic systems, such an assumption is usually not true for all noise factors. The proposed framework, by virtue of the important assumption that  $var(z_t)$  depends on the control factors, can still handle this situation by absorbing the effect of such noise factors in the disturbance term  $z_t$ . In the context of the packing experiment, temperature of the flowing material is an example of such a correlated noise factor, which can hardly be controlled and thus cannot be treated as an experimental factor in the study. However, its effect can easily be absorbed into the disturbance term  $z_t$ . Further, if such noise factors can be measured on-line, then one can think of augmenting a feedforward loop to the system. Such situations call for an integration of robust parameter design with a combination of feedback and feedforward

control and is an interesting topic for future research.

The performance measure modeling approach is a simple alternative to closed-loop estimation. However, as already discussed before, if the number of significant process variables is large, then a sequential approach may be necessary to obtain the optimal solution. Estimation of the true model from a closed-loop parameter design experiment is another worthy topic for research.

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Table 1: Factors and Levels for Response Modeling Simulation Experiment

Factor		Level		
		1	2	3
$X_1$ .	First control factor	2.2	2.3	2.4
$X_2$ .	Second control factor	3	5	7
$N$ .	Noise factor	-2	2	–
$C$ .	Adjustment factor	10	15	20

Table 2: Summarized Data from Response Modeling Simulation Experiment

Control factor		N = -2			N = 2		
$X_1$	$X_2$	$\alpha$	$g$	$\sigma^2$	$\alpha$	$g$	$\sigma^2$
2.2	3	-4.625	0.3511	0.0616	-4.182	0.3654	0.0618
2.2	5	-4.578	0.3571	0.0550	-4.178	0.3330	0.0535
2.2	7	-4.595	0.3504	0.0613	-4.238	0.3397	0.0603
2.3	3	-4.909	0.2753	0.0308	-4.309	0.2752	0.0254
2.3	5	-4.909	0.2718	0.0203	-4.330	0.2832	0.0242
2.3	7	-4.887	0.2848	0.0274	-4.287	0.2820	0.0248
2.4	3	-5.200	0.1977	0.0064	-4.366	0.1920	0.0061
2.4	5	-5.207	0.1989	0.0055	-4.391	0.1976	0.0054
2.4	7	-5.220	0.2013	0.0070	-4.405	0.1991	0.0065

Table 3: Factors and Levels for Performance Measure Modeling Simulation Experiment

Factor		Level		
		1	2	3
$X_1$ .	First control factor	2.2	2.3	2.4
$X_2$ .	Second control factor	3	5	7
$k_0$ .	Control equation constant	12	16	24
$k_I$ .	Control equation constant	1.5	3.0	4.5

Table 4: Summarized Data from Performance Measure Modeling Simulation Experiment

Factor levels				MSE from 200 observations	Factor levels				MSE from 200 observations
$X_1$	$X_2$	$k_0$	$k_I$	MSE	$X_1$	$X_2$	$k_0$	$k_I$	MSE
2.2	3	12	1.5	0.0705	2.3	5	18	3.0	0.0270
2.2	3	12	3.0	0.0704	2.3	5	18	4.5	0.0300
2.2	3	12	4.5	0.1165	2.3	5	24	1.5	0.0516
2.2	3	18	1.5	0.0741	2.3	5	24	3.0	0.0495
2.2	3	18	3.0	0.0759	2.3	5	24	4.5	0.0443
2.2	3	18	4.5	0.1173	2.3	7	12	1.5	0.0528
2.2	3	24	1.5	0.1829	2.3	7	12	3.0	0.0429
2.2	3	24	3.0	0.1590	2.3	7	12	4.5	0.0520
2.2	3	24	4.5	0.2227	2.3	7	18	1.5	0.0367
2.2	5	12	1.5	0.0720	2.3	7	18	3.0	0.0269
2.2	5	12	3.0	0.0720	2.3	7	18	4.5	0.0307
2.2	5	12	4.5	0.0963	2.3	7	24	1.5	0.0667
2.2	5	18	1.5	0.0962	2.3	7	24	3.0	0.0496
2.2	5	18	3.0	0.0806	2.3	7	24	4.5	0.0521
2.2	5	18	4.5	0.2000	2.4	3	12	1.5	0.0958
2.2	5	24	1.5	0.1469	2.4	3	12	3.0	0.0567
2.2	5	24	3.0	0.1269	2.4	3	12	4.5	0.0536
2.2	5	24	4.5	0.2304	2.4	3	18	1.5	0.0399
2.2	7	12	1.5	0.0669	2.4	3	18	3.0	0.0289
2.2	7	12	3.0	0.0653	2.4	3	18	4.5	0.0238
2.2	7	12	4.5	0.1142	2.4	3	24	1.5	0.0150
2.2	7	18	1.5	0.0804	2.4	3	24	3.0	0.0114
2.2	7	18	3.0	0.0946	2.4	3	24	4.5	0.0124
2.2	7	18	4.5	0.1168	2.4	5	12	1.5	0.0904
2.2	7	24	1.5	0.1616	2.4	5	12	3.0	0.0622
2.2	7	24	3.0	0.1435	2.4	5	12	4.5	0.0510
2.2	7	24	4.5	0.2268	2.4	5	18	1.5	0.0421
2.3	3	12	1.5	0.0492	2.4	5	18	3.0	0.0244
2.3	3	12	3.0	0.0390	2.4	5	18	4.5	0.0206
2.3	3	12	4.5	0.0485	2.4	5	24	1.5	0.0164
2.3	3	18	1.5	0.0357	2.4	5	24	3.0	0.0127
2.3	3	18	3.0	0.0315	2.4	5	24	4.5	0.0112
2.3	3	18	4.5	0.0386	2.4	7	12	1.5	0.0851
2.3	3	24	1.5	0.0611	2.4	7	12	3.0	0.0557
2.3	3	24	3.0	0.0441	2.4	7	12	4.5	0.0508
2.3	3	24	4.5	0.0528	2.4	7	18	1.5	0.0408
2.3	5	12	1.5	0.0639	2.4	7	18	3.0	0.0272
2.3	5	12	3.0	0.0403	2.4	7	18	4.5	0.0258
2.3	5	12	4.5	0.0525	2.4	7	24	1.5	0.0146
2.3	5	18	1.5	0.0312	2.4	7	24	3.0	0.0123
—	—	—	—	—	2.4	7	24	4.5	0.0120

Table 5: Factors and Levels, Packing Plant Experiment

Control Factor		Level		
		1	2	3
$X_1$ .	Sample frequency	10	20	—
$X_2$ .	Sample number	3	5	—
$X_3$ .	Sample frequency timer	240	300	—
$X_4$ .	Main feed blanking timer (sec)	0.5	0.9	—
$X_5$ .	Dribble feed blanking timer (sec)	0.5	0.9	—
$X_6$ .	Discharge timer (sec)	0.3	0.6	0.9
$X_7$ .	Dribble feed time correction constant (sec)	0.1	0.8	—
$X_8$ .	Gate allowance timer (sec)	0.4	2.0	—
$X_9$ .	Feed delay timer (sec)	0.3	0.7	—
$X_{10}$ .	Dribble feed quantity (start) (Kg)	8	12	—
$X_{11}$ .	Discharge cut-off value (Kg)	20	30	—
$X_{12}$ .	Overweight tolerance (Kg)	0.15	0.20	—
$X_{13}$ .	Underweight tolerance (Kg)	0.10	0.15	—
$X_{14}$ .	Dribble feed time (sec)	1.2	1.4	1.6
PI control scheme parameters		Level		
		1	2	3
$k_I$ .	Auto compensation proportional constant	0.2	0.3	0.4
$k_0$ .	In-flight material compensation (start)	2.0	3.0	—
Noise factors		Level		
		1	2	3
$N$ .	Composition of material	$N_1$	$N_2$	$N_3$

Table 6: Data from the Packing Experiment

Control Factor														PI		Variation over 3 noise levels	
$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$X_{10}$	$X_{11}$	$X_{12}$	$X_{13}$	$X_{14}$	$k_I$	$k_0$	s.d.	$\ln(s^2)$
+	-	+	+	+	0	-	+	-	+	+	-	+	0	1	-	0.0584	-5.6821
-	-	-	+	-	0	-	-	-	-	+	+	-	1	0	+	0.1167	-4.2966
+	+	-	-	+	0	+	+	-	+	-	+	-	0	1	+	0.1320	-4.0499
-	+	+	-	-	0	+	-	-	-	-	-	+	1	0	-	0.0979	-4.6484
+	+	+	-	+	0	+	-	+	-	+	-	-	1	1	-	0.0287	-7.1007
-	+	-	-	-	0	+	+	+	+	+	+	+	0	0	+	0.0347	-6.7234
+	-	-	+	+	0	-	-	+	-	-	+	+	1	1	+	0.1100	-4.4138
-	-	+	+	-	0	-	+	+	+	-	-	-	0	0	-	0.0289	-7.0905
-	+	+	+	-	2	-	-	-	-	+	+	+	0	1	-	0.0235	-7.4974
+	+	-	+	+	2	-	+	-	+	+	-	-	1	0	+	0.1066	-4.4772
-	-	-	-	-	2	+	-	-	-	-	-	-	0	1	+	0.1065	-4.4792
+	-	+	-	+	2	+	+	-	+	-	+	+	1	0	-	0.0421	-6.3351
-	-	+	-	-	2	+	+	+	+	+	+	-	1	1	-	0.0232	-7.5304
+	-	-	-	+	2	+	-	+	-	+	-	+	0	0	+	0.1465	-3.8414
-	+	-	+	-	2	-	+	+	+	-	-	+	1	1	+	0.1021	-4.5632
+	+	+	+	+	2	-	-	+	-	-	+	-	0	0	-	0.0206	-7.7693
+	-	-	-	-	1	-	+	-	-	+	-	-	0	1	-	0.0165	-8.2079
-	-	+	-	+	1	-	-	-	+	+	+	+	2	2	+	0.1134	-4.3529
+	+	+	+	-	1	+	+	-	-	-	+	+	0	1	+	0.0955	-4.6972
-	+	-	+	+	1	+	-	-	+	-	-	-	2	2	-	0.0130	-8.6886
+	+	-	+	-	1	+	-	+	+	+	-	-	2	1	-	0.0187	-7.9619
-	+	+	+	+	1	+	+	+	-	+	+	+	0	2	+	0.0644	-5.4851
+	-	+	-	-	1	-	-	+	+	-	+	+	2	1	+	0.1531	-3.7536
-	-	-	-	+	1	-	+	+	-	-	-	-	0	2	-	0.0255	-7.5840
-	+	-	-	+	2	-	-	-	+	+	+	+	0	1	-	0.0259	-7.3038
+	+	+	-	-	2	-	+	-	-	+	-	-	2	2	+	0.1737	-3.5010
-	-	+	+	+	2	+	-	-	+	-	-	-	0	1	+	0.1190	-4.2572
+	-	-	+	-	2	+	+	-	-	-	+	+	2	2	-	0.0222	-7.6150
-	-	-	+	+	2	+	+	+	-	+	+	-	2	1	-	0.0180	-8.0317
+	-	+	+	-	2	+	-	+	+	+	-	+	0	2	+	0.1510	-3.7810
-	+	+	-	+	2	-	+	+	-	-	-	+	2	1	+	0.0311	-6.9418
+	+	-	-	-	2	-	-	+	+	-	+	-	0	2	-	0.0178	-8.0519